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OPTICS OF BARRAQUER'S KERATOMILEUSIS

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Surgical Correction of the ocular refraction by means of surgery tending to modify the radii of curvature of the anterior corneal surface, presumes the knowledge of the exact radii of curvature that this surface should be given in order to corret a given Ametropia.

Barraquer, in one of his recent papers (1), indicates that his optical calculations are somewhat empirical and uses for them the formula Da + D taking the dioptric values from the Zeiss ophthalmometer calculations. Whereas in the anterior formula, Da stands for diopters of the anterior corneal face of the eye to be operated upon, and D for the needed correction to neutralize Ametropia.

It is known that by means of an ophthalmometer only the radius of the front surface of the cornea can be measured, whereas the total refractive power of the cornea can be read off a second scale. The refractive powers of this second scale were in a second step converted from the radii primarily measured and the result is the scale of the refractive powers. Actually the conversion requires the knowledge of further data (thickness of the cornea, curvature of the near surface of the cornea) which are not directly measured with the ophthalmometer. That means that the conversion is in fact based on a convention. I wanted to provide for a change of this convention, at least to such an extent that the correct value for the total refractive power results for a normal cornea. The imaginary refractive index $n = 1.332$ which I differentiated in my paper, served this purpose.

Considering that the ophthalmometer is an instrument that measures radius and that its relationship to diopters is only established to represent closely the total retractive value of the cornea (2) and furthermore its astigmatism, thus its application to the calculation of surgical correction of Ametropiae is subject to certain deviations that in the case of high corrections amount to a significant value.

In the following I would like to explain the development of the computation which serves to determine the exact value for the radius of the front surface of the cornea, and which must be determined in order to be able to compensate for an Ametropia by means of Keratomileusis.

The exact values shall then be compared with those computed by means of the formula $R = \frac{332}{D}$. Finally the influence of the thickness of the cornea on the result will be discussed.

For the exact computation it is assumed that the radius of curvature of the front surface of the cornea and the total thickness of the cornea (i. e. remainder of the cornea plus ground transplant of the patient's eye which is put on again) are changed. It is furthermore assumed that the curvature and the relative position of the second surface of the cornea remain unchanged compared with the other elements of the eye. That means that the depth of the anterior chamber as well as position, thickness and radii of curvature of the lens, and distance between lens and fovea remain constant. The exact computation then applis to all ametropiae, whatever their reasons may be, i. e. also to axis ametropia, refractive power ametropiae, etc. Another pre-condition is that the eye should be accommodation-free.

Fig. 1 and the following list give a survey which data of the eye are required for the computation:

n_o = refractive index of the air ($n_o = 1$).

n = refractive index of the corneal substance ($n = 1{\cdot}376$).

r_a = radius of the front surface of the uncorrected ametropic eye.

d_a = thickness of the cornea of the uncorrected ametropic eye.

r = radius of the front surface of the cornea, corrected for ametropia.

d = thickness of the cornea of the eye, corrected for emmetropia.

s'_a = distance between distant point P and vertex of the front surface of the cornea.

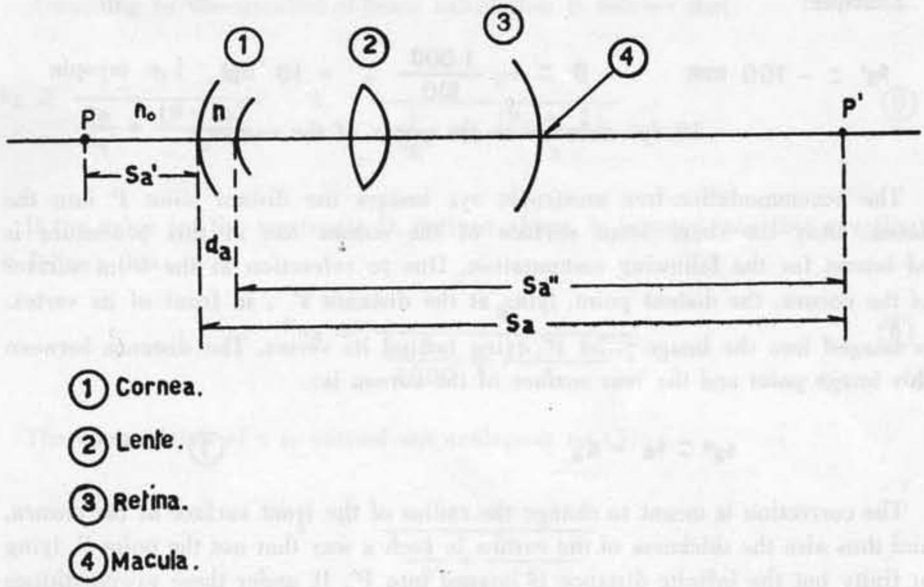

Fig 1

s_a = distance between the immage P' produced by this surface of P and the vertex of the front surface of the cornea.

$s_{''}$ = distance between image point P' and vertex of the rear surface of the cornea.

$s' = \infty$ = distance of the distant point lying at infinity from the vertex of the front surface of the cornea of the eye, corrected for emmetropia.

s = distance of the image P' produced by this surface from infinity.

s" = distance between image point P' and vertex of the rear surface of the cornea, if the eye is corrected for emmetropia.

All lengths are expressed in mm, lines which are directed towards the right in Fig. 1 are positive, those directed towards the left, are negative. Definition of ametropia:

$$D = \frac{1.000}{s_a'} \text{ dpt.}$$

Example:

$s_{a'} = -100$ mm $\quad D = -\frac{1.000}{100} = -10$ dpt., i. e. myopia

— 10 dpt. referred to the vertex of the cornea.

The accommodation-free ametropic eye images the distant point P into the fovea. Only the share front surface of the cornea has in this procedure is of interst for the following computation. Due to refraction at the front surface of the cornea, the distant point, lying at the distance s'_a, in front of its vertex, is imaged into the image point P', lying behind its vertex. The distance between this image point and the rear surface of the cornea is:

$$s_a'' = s_a - d_a \qquad (1)$$

The correction is meant to change the radius of the front surface of the cornea, and thus also the thickness of the cornea in such a way that not the point P, lying at finity but the infinite distance is imaged into P'. If under these pre-conditions all other elements of the eye remain unchanged, the eye is thus corrected for emmetropiae, since the further imaging of P' to the fovea is carried out unaltered. Analogous to (1) it follows for the corrected eye that:

$$s'' = s - d \qquad (2)$$

The demand for the correction is:

$$s'' = s_a'' \qquad (3)$$

It follows from the formulae (1), (2), and (3) that:

$$s_a = s + (d_a - d) \qquad (4)$$

That means that the mathematical task is to calculate for a given ametropia $D = \frac{1000}{s_{a'}}$ the image distance s_a and for an infinitely distant object point the image distance s, thereby determining the radius of curvature r of the corrected front surface of the cornea in such a way that the relation (4) is true.

According to the so-called 0-beam calculation it follows that:

$$s_a = \frac{n}{\frac{n_0}{s_a'} + \frac{(n-n_0)}{r_a}} = \frac{n}{\frac{1}{s_a'} + \frac{(n-1)}{r_a}} \quad (5)$$

If the value for the ametropia D, defined above, is inserted into this equation, it follows that:

$$s_a = \frac{n}{\frac{D}{1.000} + \frac{(n-1)}{r_a}} \quad (6)$$

The computation of s is carried out analogous to (5):

$$s = \frac{n}{\frac{1}{s'} + \frac{(n-1)}{r}}$$

Since for the eye, corrected for emmetropia, $s' = \infty$ and therefore $\frac{1}{s'} = 0$.

this formula is simplified to:

$$s = \frac{n \cdot r}{(n-1)} \quad (7)$$

The corrected radius r follows from the combination of the formulae (4), (5), and (7).

$$r = \frac{r_a}{\frac{r_a \cdot D}{1.000\,(n-1)} + 1} - \frac{n-1}{n}\,(d_a - d) \quad (8)$$

or inserting $n = 1.376$.

$$r = \frac{r_a}{\frac{r_a \cdot D}{376} + 1} - 0,273\ (d_a - d) \quad (9)$$

As shown by equation (9) the corrected radius r only depends on the radius r_a before correction, the ametropia D to be corrected, and the thickness difference $(d_a - d)$. Apart from the refractive index of the corneal substance $n = 1.376$ the formula does not contain any further data, in particular no data on the remaining optical system of the eye and on the reasons for the ametropia.

The following table indicates the exact radii r, calculated according to (9), as well as the radii, calculated according to the ophthalmometer formula:

$$R = \frac{332}{43,05 + D}$$

$r_a = +7,7\,mm$ and $(d_a - d) = 0,3\,mm$. are assumed as an example.

The last column of the table shows the difference between the exactly and the approximately computed radii.

TABLE I

	Ametropia	Exact radius r	$R = \frac{332}{D}$	r - R
Hypermetropia	+ 20 dpt + 10 dpt	5,381mm 6,309mm	5,265mm 6,258mm	+ 0,116mm + 0,051mm
Myopia	— 10 dpt — 20 dpt	9,601mm 12,960mm	10,045mm 14,403mm	— 0,444mm — 1,443mm

It is even more interesting if the ametropiae. D, compensated by the exactly computed radius r, is compared with D_R which is compensated if the approximately computed radius R is entered into the formula (9) instead of r.

After expansion to D it follows from the formula (9):

$$D = 1.000\,(n-1)\left\{\frac{1}{r + \frac{n-1}{n}(d_a - d)} - \frac{1}{r_a}\right\}\text{dpt} \qquad (10)$$

or

$$D = 376\left\{\frac{1}{r + 0,273\,(d_a - d)} - \frac{1}{r_a}\right\}\text{dpt} \qquad (10)$$

As an example I take $r_a = 7.7$ mm and $(d_a - d) = 0.3$ mm and that r is replaced by the R — values of Table 1, it follows that:

$$D_R = 376 \left\{ \frac{1}{R + 0{,}273 \cdot 0\,3} - \frac{1}{7{,}7} \right\} \text{dpt}$$

The result is:

TABLE II

D	D_R	$D\text{-}D_R$
+ 20 dpt	+ 21,5 dpt	— 1,5 dpt
+ 10 dpt	+ 10,5 dpt	— 0,5 dpt
+ 5 dpt	+ 4,9 dpt	+ 0,1 dpt
— 5 dpt	+ 6,1 dpt	+ 1,1 dpt
— 10 dpt	— 11,7 dpt	+ 1,7 dpt
— 20 dpt	— 22,9 dpt	+ 2,9 dpt

The last column of Table II indicates the difference between the ametropia D, which is to be corrected and the ametropia D_R which would be compensated for in this example by the radius R. Negative sign means in this example that the eye corrected by means of R is not emmetrop but myopic, and the positive sign that it becomes hypermetropic.

According to the result, given in Talbe II, it is recommendable to calculate the desired radius r, according to formula (9).

The influence of the corneal thickness before and after the operation is furthermore of interest, since both thicknesses are included in formulae (9) and (10). If in formula (10) D is considered as a function of the difference in thickness: $\delta = (d_a - d)$, and if the differential quotient is formed to this difference, it follows that:

$$\Delta D = \frac{\left(D + \frac{1.000\ (n-1)}{r_a}\right)^2}{1{,}000 . n} . \Delta\delta = \frac{\left(D + \frac{376}{r_a}\right)^2}{1.376} . \Delta\delta \quad (11)$$

Particulary.

for r_a =7.7 mm. it follows that:

$$\Delta D = \frac{(D + 48,8)^2}{1.376} \cdot \Delta\delta \text{ dpt} \qquad (12)$$

A change in the difference in thicness $\Delta\delta$ results in a correction error ΔD. Table III indicates the influence of an error in the difference in thickness of $\Delta\delta =$ 0,1 mm. for different ametropiae:

TABLE III

D	ΔD for $\Delta\delta = 0{,}1$ mm
+ 20 dpt	+ 0,34 dpt
+ 10 dpt	+ 0,25 dpt
+ 5 dpt	+ 0,21 dpt
— 5 dpt	+ 0,14 dpt
— 10 dpt	+ 0,11 dpt
— 20 dpt	+ 0,06 dpt

I do hope that the results given herein shall be of assistance to the surgeons and useful for further research work, and I would be glad if they stood as a contribution to a still more favorable correction by surgical means to the refractive errors of the eye.

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