Arch. Soc. Amer. Oftal. Optom. (1970) - 8 - 165

THE OPTICS INVOLVED IN CORRECTING AMETROPIC EYES WITH THE AID OF KERATOMILEUSIS AND KERATOPHAKIA *

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The methods of surgical correction of ametropic eyes developed to great perfection by Dr. José I. Barraquer require exact knowledge of the essential optical properties of the eye. These optical conditions will here be explained in an easily understandable manner and without going into an unnecessary sources of error and their effect on the result of the surgical correction. Finally, the aphakic eye will be discussed in particular.

Keratomileusis and keratophakia serve to vary the curvature of the anterior corneal surface and the thickness of the cornea in order to compensate an existing ametropia. Let us assume that the surgical method as such is known [1]. There is no basic difference between the two methods as far as optics arec concerned. For each of the methods we have to determine the modification of the cornea required to make the affected eye emmetropic. A mathematically exact answer to this question has already been given [2]. The present paper is intended to explain the underlying idea of the computation, but not the computation proper.

Fig. 1 shows the elements of the eye that are essential for optical image formation, a myopic eye having been chosen as an example. With relaxed accommodation, the remote point P is imaged on the fovea P_J by the entire optical system of the eye. In this process, the image-forming light rays are refracted by four surfaces. The first refraction takes place at the front curvature of the cornea so that the ray which in Fig. 1 emanates from P is directed at the point P_1 in the interior of the cornea after

* Lecture delivered at the V International Course of Ophthalmology. Instituto Barraquer. Barcelona. May 1969.

passing through the aforementioned surface. P_1 is the image of P formed by the anterior corneal surface. The position of P_1 is affected by:

1. The radius r of the front surface of the cornea;
2. the refractive index n_1 of the substance of the refracting surface (in the present case, air $n_1 = 1$),
3. the refractive index n_2 of the corneal substance,
4. the distance from the far point P to the vertex of the cornea S_1.

After the ray has covered the distance d in the cornea, it is refracted a second time by the back surface of the cornea. After this refraction the ray proceeds in the interior of the anterior chamber towards the second image point P_2. This new direction is determined by the radius of the posterior corneal surface and the refractive index n_3 of the aqueous as well as the refractive index n_2 of the corneal substance. The following two changes in direction occur at the front surface (image point P_3) and the back surface (image point P_4) of the lens. P_4 lies on the fovea of the eye.

P_1, P_2, P_3 and P_4 are optically conjugated images. An image formed by the front surface of the cornea at P_1 is automatically passed on via P_2 and P_3 to P_4 and imaged on the fovea. To correct an ametropic eye it is therefore sufficient to vary the curvature and thickness of the anterior corneal surface so that it is not P but infinity, i.e. a ray incident on the cornea parallel to the optical axis, that is imaged at P_2 (Fig. 2). In the present example of a myopic eye, this could be achieved by flattening the front surface of the cornea accordingly, i.e. by increasing its radius.

The computation is considerably simplified by the fact that the image of of P_1 at P_4 need not be known for the calculation. However, this requires that the curvature of the posterior corneal surface and the location and curvature of the lens remain unchanged. The following formula (1) indicates the radius r_c which the front surface of the cornea must acquire in order to correct the ametropia of the eye. This formula applies exactly for any type of ametropia, i.e. both for axial ametropia and refractive ametropia as for aphakic eyes. An important fact in its application is that the formula contains only quantities that can easily be measured or, like n_2, are known with sufficient accuracy. The formula, derived earlier [2], reads:

$$r_c = \frac{r}{\dfrac{r \cdot D}{1000\,(n_2 - 1)} + 1} - \frac{n_2 - 1}{n_2}\,(d - d_c). \qquad (1)$$

were

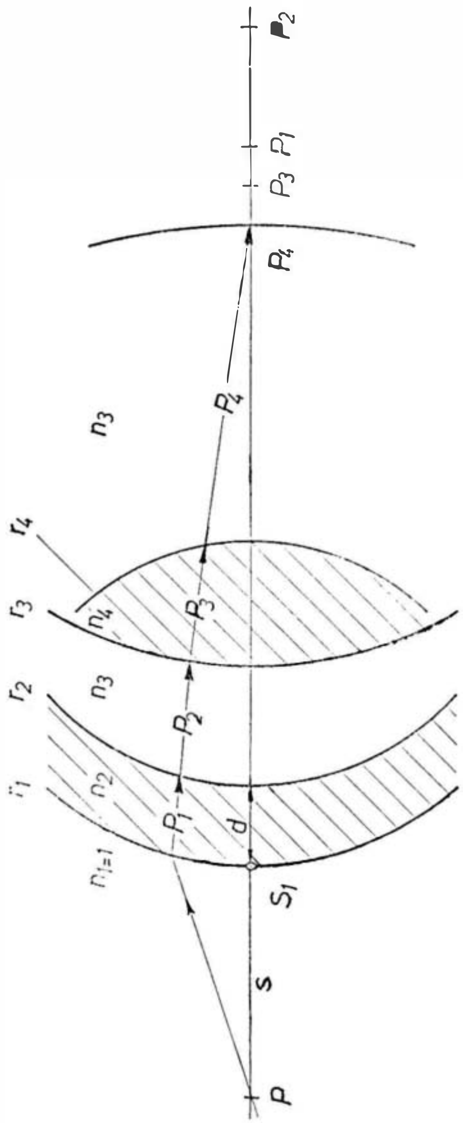

Figura No. 1

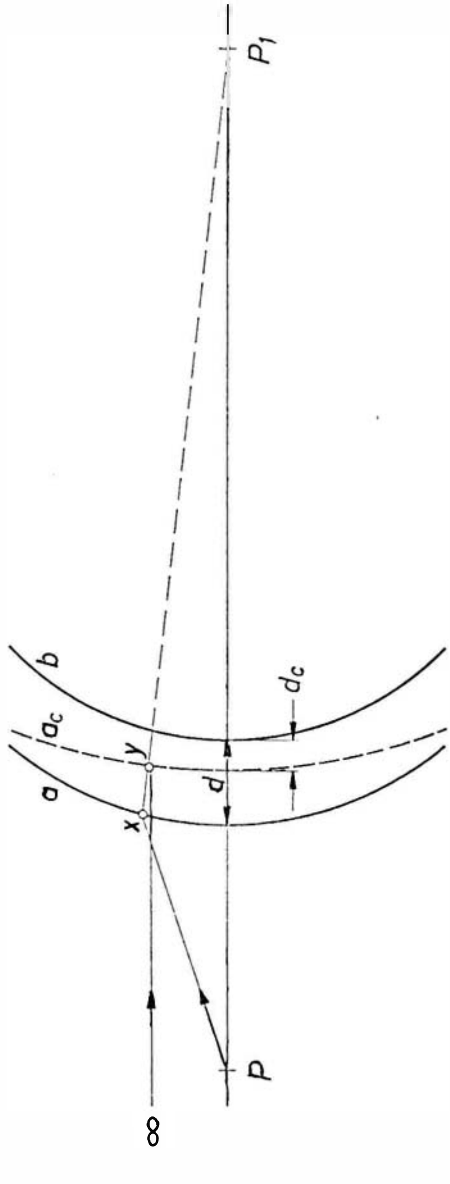

Figura No. 2

r = radius of the anterior corneal surface of the ametropic eye before surgery

cr = radius of the anterior corneal surface after surgical correction to obtain emmetropia

d = thickness of the cornea before surgery

dc = thickness of the cornea after surgical correction to obtain emmetropia

n_2 = refractive index of corneal substance

D = ametropia to be corrected.

As usual, D is defined as the reciprocal distance s of the far point from the vertex of the cornea. If the far point lies in front of the vertex S_1 as in Fig. 1, i.e. if the lines s from the vertex to the far point runs from right to left, then s is negative.

Example: $s = -100$ mm. $D = \frac{1000}{s} = \frac{1000}{-100} = -10$ dpt myopia.

In keratomileusis, dc results more or less automatically from the method of operation. In keratophakia, dc is essentially determined by the thickness of the transplanted cornea. If dc is known, the radius rc to be generated by surgery can be determined with the aid of (1).

The success of the operation is largely dependent on how accurately the values rc and dc determined by (1) can be obtained. However, it also depends on the accuracy which the other magnitudes of (1) are known. If these magnitudes are not known exactly, it will be impossible to correct the ametropia D completely. A residual ametropia ΔD will remain. In the following, the effect of these errors on the result of the computation according to formula (1) will be discussed.

1. **Varying r**

An error Δr in the measurement of the anterior corneal surface's radius of curvature will cause a residual amount of ametropia after surgery of

$$\Delta D = -\frac{1000\ (n_2 - 1)}{r^2} \cdot \Delta r \text{ dpt} \qquad (2)$$

Example: $r = 7{,}7$ mm. $n = 1.376$ $\Delta D = -6.3\ \Delta r$ dpt
A measuring error of $\Delta r = \pm 0.02$ mm. thus gives rise to a residual ametropia of $\Delta D \pm 0.13$ dpt.

This accuracy can be obtained by careful ophthalmometric measurement so that errors of correction exceeding approx. 0.2 dpt need hardly be expected as a result of erroneous determination of radii.

2. **Varying d — dc**

Formula (1) contains the difference in corneal thickness before and after correction. Consequently, an and error in the measurement of both thickness difference will also affect the correction, with.

$$\Delta D = \frac{\left(D + \frac{1000\ (n_2 - 1)}{r}\right)^2}{1000\ .\ n_2}\ .\ \Delta\ (d - dc)\ \text{dpt} \qquad (3)$$

being the residual ametropia after correction.

Example: r = 7.7 mm. Δ (d — dc) = 0.1 n_2 = 1.376

D	Δ D
+ 20 dpt	+ 0.34 dpt
+ 10	+ 0.25
— 10	+ 0.11
— 20	+ 0.06

This right-hand column of the table indicates the residual ametropia caused by an error of 0.1 mm. in determining the thickness difference. This table clearly shows that accurate measurement of thickneses is indispensables, above all when correcting hypermetropic eyes.

3. **Varying n_2**

The refracting power of the corneal substance is not measured in every single case. Instead, an average value of n_2 = 1.376 is assumed. If the refractive index deviates from this value by Δn_2, a residual ametropia of

$$\Delta D = -\frac{\Delta n_2}{n_2 - 1}\ .\ D = -\frac{\Delta n_2}{0.376}\ .\ D \qquad (4)$$

will remain after correction. For example, if the corneal substance has a refractive index of 1.386 instead of 1.376, then Δn_2 = + 0.01. This gives the residual ametropia listed in the right-hand column of the table below.

D	Δ D
+ 20 dpt	— 0.53 dpt
+ 10	— 0.27
— 10	+ 0.27
— 20	+ 0.53

As a result, hypermetropic eyes will be slightly myopic after surgery, and myopic eyes slightly hypermetropic.

According to earlier publications [3], it may be assumed that the refractive index of the cornea will not deviate from the standard value by more than $\Delta n = + 0.004$. The residual ametropia is then less than half the values given.

4. **Varying r_2**

The radius r_2 of the posterior corneal surface is not contained in formula (1) since this formula is based on the assumption that the curvature of the the back surface of the cornea is not changed by the operation. In spite of this, however, it may be useful to study the effect which a possible variation of r_2 would have on the correction. Since the refractive index of the cornea varies only slightly from that of the aqueous, a variation of r_2 will have only a minor effect on the correction, viz. an average of 1/9 of the amount which would be introduced by a variation of the curvature of the anterior corneal surface.

Example: if the radius of the anterior corneal surface of an emmetropic eye is increased by 0.1 mm., the eye will become hypermetropic by 0.63 dpt. By comparison, an increase in the radius of the back surface of the cornea by likewise 0.1 mm. will result in a hypermetropia of only 0.07 dpt. It will thus actualliy be possible to neglect the posterior corneal surface.

The aphakic eye

The above statements —including formula (1)— apply without restriction to the aphakic eye as well. After removal of the lens, we obtain a hypermetropia of $D = + 12.9$ dpt. as standard value for the originally emmetropic Gullstrand's eye. To correct this, the radius of the anterior corneal surface would have to be reduced from 7.7 mm. to 6.1 mm., which is practically impossible without implanting corneal substance. As is known, this is one of the reasons why keratophakia was introduced.

A special situation results if the ametropia of an aphakic eye is to be corrected by keratophakic surgery, in which case the corneal data required for substitution in formula (1) cannot be computed because the ametropia of the eye cannot be measured as long as it contains the turbid lens.

Since the aphakic eye is a very simple optical system, the raddi of curvature r_1 and r_2 and the refractive index of the cornea alone suffice to compute the ametropia which the eye will have after removal of the lens.

provided that the axial distance 1, i.e. the distance from the fovea to the front vertex of the cornea, is known. As reported by Gernet[4], the axial distance can be measured to within approx. + 0.3 mm. to $\pm$ 0.5 mm. by ultrasonic means.

The exact computation of ametropia making allowance for refraction by the two corneal surfaces can in this case be replaced by a simplified calculation in which only the front surface of the cornea is taken into account, provided that a substance with a refractive index $\tilde{n} = 1.331$ is assumed behind this surface. The simplified formula can be written as:

$$D = 1000 \left(\frac{\tilde{n}}{1} = \frac{(\tilde{n} - 1)}{r}\right)$$

or, if the value ñ is substituted,

$$D = \frac{1331}{1} - \frac{331}{r} \text{ dpt} \qquad (5)$$

In this formula, the axial distance 1 and the radius r of the front surface of the cornea must be (substituted) given in millimeters. Use of this simple formula is permissible because it determines ametropia over the entire practical range to within $\pm$ 0.04 dpt —an error that is negligible compared with the effect of the uncertainty of 1.

Now there remains to be seen to what extent a measuring error $\Delta 1$ of the axial distance 1 will affect the computed ametropia D. The error ΔD of ametropia is

$$\Delta D = - \frac{\left(D + \frac{331}{r}\right)^2}{1331} \cdot \Delta 1 \text{ dpt} \qquad (6)$$

if we assummer $r = 7{,}7$, then a measuring error of $\Delta 1 = \pm 0.5$ mm. will give rise to the errors of ametropia given in the right-hand column of the table below.

D	Δ D
+ 20 dpt	— 1.5 dpt
0	— 0.7
— 20	— 0.2

Summary

Keratomileusis and keratophakia serve to correct ametropic eyes by varying the curvature of the front surface and the thickness of the cornea.

The same exact formula holds for each of the two methods. This formula contains only easily measurable or known quantities.

A detailed discussion of errors shows the residual ametropia caused by errors in the measured quantities.

Finally, correction of the aphakic eye is discussed, in which the degree of ametropia cannot be optically determined prior to surgery due to the turbidity of the lens.

The degree of ametropia can in this case be computed if the exial distance is determined by ultrasonic means. Possible errors are discussed as well.

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